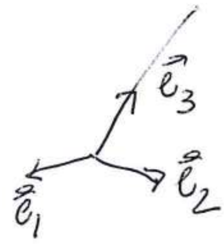
Buckling of twisted rod



$\vec{e}_i$ - material orts

$\vec{e}_3$ - tangent

$$\frac{d\vec{e}_i}{dl} = \vec{\Omega} \times \vec{e}_i$$

$$\vec{M} = A(\Omega_1 \vec{e}_1 + \Omega_2 \vec{e}_2) + C\Omega_3 \vec{e}_3$$

$$\frac{d^2\vec{r}}{dl^2} = \vec{\Omega} \times \frac{d\vec{r}}{dl} \qquad \Big| \frac{d\vec{r}}{dl} \times$$

$$\Omega_1 \vec{e}_1 + \Omega_2 \vec{e}_2 = \frac{d\vec{r}}{dl} \times \frac{d^2\vec{r}}{dl^2}$$

$$\frac{d\vec{M}}{dl} + [\vec{e}_3 \times \vec{F}] = 0$$

$$\frac{d\vec{F}}{dl} = 0 \Rightarrow \vec{F} = \vec{F}_0$$

$$A \frac{d\vec{r}}{dl} \times \frac{d^3\vec{r}}{dl^3} + C\Omega_3 \frac{d^2\vec{r}}{dl^2} + \frac{d\vec{r}}{dl} \times \vec{F}_0 = 0$$

! Ω_3 = const

Illustration

$$0 = \vec{e}_3 \frac{d\vec{M}}{dl} = C\frac{d\Omega_3}{dl} + A\Omega_1 \vec{e}_3 \frac{d\vec{e}_1}{dl} + A\Omega_2 \vec{e}_3 \frac{d\vec{e}_2}{dl} =$$

$$= C\frac{d\Omega_3}{dl} + A\Omega_1 \vec{e}_3 [\vec{\Omega} \times \vec{e}_1] + A\Omega_2 \vec{e}_3 [\vec{\Omega} \times \vec{e}_2] =$$

$$= C\frac{d\Omega_3}{dl} + A\Omega_1 \vec{\Omega} [\vec{e}_1 \times \vec{e}_3] + A\Omega_2 \vec{\Omega} [\vec{e}_2 \times \vec{e}_3] =$$

$$= C\frac{d\Omega_3}{dl} - A\Omega_1\Omega_2 + A\Omega_2\Omega_1 = C\frac{d\Omega_3}{dl}$$

Stability of

$\vec{l}_3^{\,0} \uparrow \quad \uparrow \vec{F}_0 \qquad \frac{d\vec{r}^{\,0}}{dl} = (0,0,1) \qquad \vec{F}_0 = (0,0,F_0)$

Neutral solution for small perturbation

$$\delta\vec{r} = (x, y, 0)$$

$$A\,\vec{l}_3^{\,0} \times \frac{d^3\delta\vec{r}}{dl^2} + C\Omega_3 \frac{d^2\delta\vec{r}}{dl^2} + \frac{d\delta\vec{r}}{dl} \times \vec{F}_0 = 0$$

$$-A\frac{d^3y}{dl^3} + C\Omega_3\frac{d^2x}{dl^2} + \frac{dy}{dl}F_0 = 0$$

$$A\frac{d^3x}{dl^3} + C\Omega_3\frac{d^2y}{dl^2} - \frac{dx}{dl}F_0 = 0$$

$$\zeta = \frac{dx}{dl} + i\frac{dy}{dl}$$

$$A\frac{d^2\zeta}{dl^2} - iC\Omega_3\frac{d\zeta}{dl} - \zeta F_0 = 0$$

$$\zeta \sim e^{iml}$$

$$-Am^2 + C\Omega_3 m - F_0 = 0$$

$$m_{1,2} = \frac{C\Omega_3 \pm \sqrt{(C\Omega_3)^2 - 4F_0A}}{2A}$$

$$\zeta = De^{im_1 l} + Ee^{im_2 l}$$

Unclamped ends $\vec{l}_3^{\,0} \times \frac{d^2\delta\vec{r}}{dl^2}\Big|_{0,L} = 0$

$$\frac{d\zeta}{dl}\Big|_{0,L} = 0$$

$$i m_1 \mathcal{D} + i m_2 E = 0$$

$$i m_1 e^{i m_1 L} \mathcal{D} + i m_2 e^{i m_2 L} E = 0$$

$$\begin{vmatrix} i m_1 & i m_2 \\ i m_1 e^{i m_1 L} & i m_2 e^{i m_2 L} \end{vmatrix} = 0 \Rightarrow$$

$$m_1 m_2 \left(e^{i m_1 L} - e^{i m_2 L}\right) = 0$$

$$m_1 m_2 = \frac{F_0}{A} \neq 0$$

$$e^{i m_1 L} = e^{i m_2 L}$$

$$(m_1 - m_2) L = 2\pi n \qquad n = 1$$

$$L \frac{\sqrt{(C\Omega_3)^2 - 4F_0 A}}{A} = 2\pi$$

$$(C\Omega_3)^2 = \left(\frac{2\pi A}{L}\right)^2 + 4F_0 A$$

Buckling of untwisted rod under compression ($F_0 < 0$) $F_0 = -F_c$

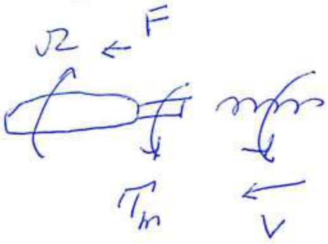
$$F_c = \frac{\pi^2 A}{L^2}$$ (see solution with free ends)

Buckling of twisted rod

$$T_c = (C\Omega_3)_c = \frac{2\pi A}{L}$$

$$\frac{F}{F_c} + \left(\frac{T}{T_c}\right)^2 = 1$$

Switching of Vibrio alginolyticus.



$$\left.\begin{aligned} -AV + B\omega - F &= 0 \\ BV - \mathcal{D}\omega + T_m &= 0 \end{aligned}\right\} \text{flagella}$$

$$\left.\begin{aligned} -\mathcal{D}_0 \Omega - T_m &= 0 \\ -A_0 V + F &= 0 \end{aligned}\right\} \text{body}$$

Solution

$$T_m = \frac{\mathcal{D}(A + A_0) - B^2}{B} V$$

$$F = A_0 V$$

Relation for the propulsion matrix

$$A = \frac{l}{\cos\psi}\left(\zeta_\parallel \cos^2\psi + \zeta_\perp (1 - \cos^2\psi)\right)$$

$$B = \frac{l}{\cos\psi}\,\frac{P}{2\pi}\,\frac{(1 - \cos^2\psi)}{\cancel{\cos\psi}}\left(\zeta_\perp - \zeta_\parallel\right)$$

$$\mathcal{D} = \frac{l}{\cos\psi}\left(\frac{P}{2\pi}\right)^2 \frac{(1 - \cos^2\psi)}{\cos^2\psi}\left(\zeta_\parallel \sin^2\psi + \zeta_\perp \cos^2\psi\right)$$

$$\cos\psi = \frac{P/2\pi}{\sqrt{R^2 + (P/2\pi)^2}}\ ; \quad \zeta_\perp = 2\zeta_\parallel$$

$$\zeta_\parallel = \frac{2\pi\eta}{\left(\ln\frac{2P}{r} - \frac{1}{2}\right)}$$

$$A_0 \, \cancel{d_R} = 6\pi\eta b \left(1 - \frac{1}{5}\left(1 - \frac{a}{b}\right)\right)$$

$$d_0 \, \cancel{B_R} = 8\pi\eta b^3 \left(1 - \frac{3}{5}\left(1 - \frac{a}{b}\right)\right)$$

Parameters of Vibrio

$a = 1.6\ \mu m$; $b = 0.6\ \mu m$ body

$$\left.\begin{array}{l} p = 1.49\ \mu m \\ r = 16\ nm \\ R = 140\ nm \\ l = 4.59\ \mu m \end{array}\right\} \text{flagella}$$

$L = 100\ nm$ hook

Bending modulus of hook

$K_b = 1.3 \cdot 10^{-26}\ N \cdot m^2$

velocity $V = 47\ \mu m/s$.

From solution of elasticity problem

$$F_c = \frac{\pi^2 K_b}{L^2}; \quad T_c = \frac{2\pi K_b}{L}$$

Neutral curve $F/F_c + \left(T/T_c\right)^2 = 1$

Estimate for Vibrio

$$F/F_c + \left(T/T_c\right)^2 \cong 0.67 \text{ - quite close}$$