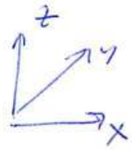
Pullez



$$T_z^{v} = D \cdot \omega$$

$$-\alpha \frac{d\vartheta}{dt} + D\omega + m H_0 \sin\vartheta = 0$$

$$K = D\omega$$

$$\tau = \frac{K}{\alpha} t$$

$$\frac{d\vartheta}{dt} = 1 + \frac{m H_0}{K}\sin\vartheta \qquad \frac{d\vartheta}{dt} > 0$$

$$2\pi = T + \beta\int_0^{2\pi}\frac{\sin\vartheta\, d\vartheta}{1+\beta\sin\vartheta}$$

$$T = \int_0^{2\pi}\left(1 - \frac{\beta\sin\vartheta}{1+\beta\sin\vartheta}\right)d\vartheta = \int_0^{2\pi}\frac{d\vartheta}{1+\beta\sin\vartheta}$$

$$z = e^{i\vartheta} \qquad dz = iz\,d\vartheta$$

$$T = \oint_{|z|=1}\frac{dz}{iz\left(1+\frac{\beta}{2i}\left(z-\frac{1}{z}\right)\right)} = \int\frac{dz}{\frac{\beta}{2}z^2 - \frac{\beta}{2} + iz} =$$

$$= \frac{1}{\beta/2}\oint\frac{dz}{z^2-1+\frac{2iz}{\beta}}$$

$$z_{1,2} = -\frac{i}{\beta} \pm \sqrt{-\frac{1}{\beta^2}+1} =$$

$$= -\frac{i}{\beta} \pm i\sqrt{\frac{1}{\beta^2}-1} \qquad \beta < 1$$

$$z_1 = -\frac{i}{\beta} + i\sqrt{\frac{1}{\beta^2}-1} \qquad |z_1| < 1$$

$$z_1 z_2 = \left(-\frac{i}{\beta} + i\sqrt{\frac{1}{\beta^2}-1}\right)\left(-\frac{i}{\beta} - i\sqrt{\frac{1}{\beta^2}-1}\right) =$$

$$= -\left(-\frac{1}{\beta} + \sqrt{\frac{1}{\beta^2}-1}\right)\left(-\frac{1}{\beta} - \sqrt{\frac{1}{\beta^2}-1}\right) =$$

$$= -\left(\frac{1}{\beta^2} - \left(\frac{1}{\beta^2}-1\right)\right) = -1$$

$$T = \frac{1}{\beta/2} \oint \frac{dz}{(z-z_1)(z-z_2)} = \frac{2\pi i}{\beta/2} \frac{1}{(z_1-z_2)}$$

$$z_1 - z_2 = 2i\sqrt{\frac{1}{\beta^2}-1}$$

$$T = \frac{\pi}{\beta/2 \sqrt{\frac{1}{\beta^2}-1}} = \frac{2\pi}{\sqrt{1-\beta^2}}$$

Mean velocity

$$\langle V_y \rangle = \frac{v_0}{T}\int_0^T \sin\vartheta\, dt = \frac{v_0}{T}\int_0^{2\pi} \frac{\sin\vartheta\, d\vartheta}{1+\beta\sin\vartheta} =$$

$$= \frac{v_0}{T}\cdot\frac{1}{\beta}\int_0^{2\pi} \frac{\beta\sin\vartheta + 1 - 1}{1+\beta\sin\vartheta}\, d\vartheta$$ и интеграл

сводится к предыдущему

$$= \frac{v_0}{T\beta}\left(2\pi - \int_0^{2\pi}\frac{d\vartheta}{1+\beta\sin\vartheta}\right) =$$

$$= \frac{v_0}{\beta}\left(\frac{2\pi}{T} - 1\right) = \frac{v_0}{\beta}\left(\sqrt{1-\beta^2} - 1\right) \qquad \beta \to 0$$

$$\langle v_y \rangle = -\frac{v_0 \beta}{2}$$

Более простой вывод для fisher.

$$\langle V_y \rangle = \frac{v_6}{T} \int_0^{-2\pi} \frac{\sin\vartheta \, d\vartheta}{\beta \sin\vartheta - 1} = \frac{v_6}{T\beta} \int_0^{-2\pi} \frac{\beta \sin\vartheta + 1 - 1}{\beta \sin\vartheta - 1} d\vartheta =$$

$$= \frac{v_6}{T\beta} \left(-2\pi + \int_0^{-2\pi} \frac{d\vartheta}{\beta \sin\vartheta - 1} \right) =$$

$$= \frac{v_6}{\beta} \left(1 - \frac{2\pi}{T} \right) = \frac{v_6}{\beta} \left(1 - \sqrt{1-\beta^2} \right)$$